

KASR TARTIBLI DIFFUZIYA TENGLAMASI FUNDAMENTAL YECHIMI

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Chust tuman 2-son Texnikumi matematika fani o'qituvchisi.

Annotatsiya: Ushbu maqolada kasr tartibli diffuziya tenglamasi fundamental yechimi kasr tartibli differensial tenglama uchun Koshi masalasi yechimi yordamida yechish masalasi hamda bu yechimning klassik yechim bo'lish masalasi o'rganilgan.

Kalit so'zlar: Kasr tartibli diffuziya tenglamasi, Koshi masalasi, Kaputo ma'nosidagi kasr tartibli hosila, Mittag-Leffler funksiyasi, Dirakning delta funksiyasi.

Kirish

Hozirgi zamon matematikasida yangi yo'nalish hisoblangan kasr tartibli hosila, kasr tartibli integral va kasr tartibli differensial tenglamalarni o'rganish dolzarb masala hisoblanadi. Bu mavzuda dunyo olimlari hozirgi kunda juda ko'p izlanishlar olib borishmoqda. Xususan respublikamizda "Romanovskiy" nomidagi matematika instituti Buxoro filialida ustozimiz prof.D.Q.Durdiyev rahbarligida bu mavzu keng o'rganilib kelmoqda.

Ushbu maqolada kasr tartibli differensial tenglamaning kasr tartibli diffuziya tenglamasini yechishdagi tadbqiq nazariy jihatdan o'rganilgan.

Masalaning qo'yilishi: Bizga bir jinsli bo'lmagan kasr tartibli diffuziya tenglamasi berilgan va bu differensial tenglamani kasr tartibli differensial tenglama uchun Koshi masalasi yordamida yechish masalasi qo'yilgan.

$$\begin{cases} {}^K D_{0+t}^\alpha y(x) + \lambda y(x) = f(x) & x \in R, t > 0 \\ y(0) = y_0 \end{cases} \quad (1) \rightarrow \text{Koshi masalasi. [3]}$$

$$y(x) = y_0 E_\alpha(-\lambda x^\alpha) + \int_0^x y^{\alpha-1} E_{\alpha,\alpha}(-\lambda y^\alpha) f(x-y) dy \quad (2) \rightarrow \text{Koshi masalasi yechimi. [3]}$$

Kasr tartibli diffuziya tenglamasining fundamental yechimini Koshining (1) masalasi yechimi yordamida keltirib chiqaramiz.

Quyidagi masala qo'yilgan:

$$\begin{cases} {}^K D_{0+t}^\alpha U(x, t) - U_{xx} = f(x, t) & x \in R, t > 0 \\ U(x, 0) = \varphi(x) \end{cases} \quad (3)$$

(3) Tenglamaning yechimini topishda Fur'e integral almashtirishlaridan foydalanamiz. [1]

f funksiyaning R dagi Fur'e almashtirishi:

$$F[f] = F[f(x)](\xi) = \hat{f}(\xi) := \int_{-\infty}^{+\infty} e^{i\xi x} f(x) dx, \xi \in R \quad \text{ko'rinishida bo'ladi.}$$

Kaputo ma'nosidagi kasr tartibli hosila uchun Fur'e integral almashtirishini qo'llaymiz:

$$F[{}^K D_{0+t}^\alpha U] = {}^K D_{0+t}^\alpha \hat{U}$$

U_{xx} ni ham Fur'e integral almashtirishi orqali ifodalaymiz:

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$$\begin{aligned} F[U_{xx}] &= \int_{-\infty}^{+\infty} e^{ixz} U_{xx} dx = \int_{\infty}^{-\infty} e^{-ixz} d(U_x) = \\ &= e^{-ixz} U_x |_{-\infty}^{\infty} + iz \int_{+\infty}^{-\infty} e^{-ixz} U_x dx = 0 + iz \int_{-\infty}^{\infty} e^{-ixz} dU = iz(e^{ixz} U |_{-\infty}^{\infty} + \\ &+ iz \int_{\infty}^{-\infty} e^{ixz} U dx = iz(0 + iz\hat{U}) = -z^2 \hat{U} \end{aligned}$$
$$F[f(x,t)](\mathfrak{z}) = \hat{f}(\mathfrak{z}, t)$$
$$\begin{cases} {}^K D_{0+t}^\alpha \widehat{U} + \mathfrak{z}^2 \widehat{U} = \widehat{f}(\mathfrak{z}, t) & \mathfrak{z} \in R, t > 0 \\ \widehat{U}(\mathfrak{z}, 0) = \widehat{\phi}(\mathfrak{z}) \end{cases} \quad (4)$$

$y(x) = y_0 E_\alpha(-\lambda x^\alpha) + \int_0^x y^{\alpha-1} E_{\alpha,\alpha}(-\lambda y^\alpha) f(x-y) dy$ ifoda (1) tenglama yechimi hisoblanadi.

$$\widehat{U}(\mathfrak{z}; t) = E_{\alpha}(-\mathfrak{z}^2 t^{\alpha}) + \int_0^t y^{\alpha-1} E_{\alpha, \alpha}(-\mathfrak{z}^2 y^{\alpha}) \widehat{f}(\mathfrak{z}; t-y) dy \quad (5)$$

Bu tenglikka ko'ra
$$\begin{cases} {}^K D_{0+t}^\alpha U(x,t) - U_{xx} = f(x,t) & x \in \mathbb{R}, t > 0 \\ U(x,0) = \varphi(x) \end{cases} \quad (3)$$

$$F^{-1}[f] = F^{-1}[f(x)](z) = \frac{1}{2\pi} \int_B e^{ixz} f(z) dz$$

Demak :

$$U = F^{-1}[\widehat{U}] = \frac{1}{2\pi} \int_R e^{ix\zeta} \widehat{u}(\zeta; t) d\zeta \quad (6)$$
$$U(x, t) = \frac{1}{2\pi} \int_R e^{ix\zeta} \left[E_\alpha(-\zeta^2 t^\alpha) + \int_0^t y^{\alpha-1} E_{\alpha,\alpha}(-\zeta^2 y^\alpha) \hat{f}(\zeta; t-y) dy \right] d\zeta \quad (7)$$
$$\begin{cases} {}^K D_{0+t}^\alpha U(x,t) - U_{xx} = f(x,t) & x \in R, t > 0 \\ U(x,0) = \delta(x) \end{cases} \quad (8)$$

$$\int_R \delta(x - x_0) \varphi(x) dx = \varphi(x_0) \quad (9)$$

$\delta(x) = \delta(-x)$ (juft funksiya)

(7) tenglik bo'yicha $U(x, 0)$ ni topamiz:

$$U(x, 0) = \frac{1}{2\pi} \int_R e^{ixz} \left[E_\alpha(-z^2 t^\alpha) + \int_0^0 y^{\alpha-1} E_{\alpha,\alpha}(-z^2 y^\alpha) \hat{f}(z; 0 - y) dy \right] dz =$$

$$\frac{1}{2\pi} \int_R e^{ixz} dz = \delta(x)$$

(bu natija $\int_0^0 y^{\alpha-1} E_{\alpha,\alpha}(-z^2 y^\alpha) \hat{f}(z; 0 - y) dy = 0$,

$E_\alpha(-z^2 t^\alpha) = \sum_{n=0}^{\infty} \frac{(-z^2 t^\alpha)^n}{\Gamma(\alpha n + 1)}$ ifodalarga ko'ra kelib chiqdi)

(9) formulaga ko'ra (7) tenglik bilan ifodalangan yechimning (8) ko'rinishidagi masala uchun o'rinli ekanligi kelib chiqadi:

$$U(x, 0) = \int_R \varphi(y) \left[\frac{1}{2\pi} \int_R e^{iz(x-y)} dz \right] dy = \int_R \varphi(y) \delta(x - y) dy = \varphi(x)$$

Bunga ko'ra

$$\begin{cases} {}^K D_{0+t}^\alpha U(x, t) - U_{xx} = f(x, t) & x \in R, t > 0 \\ U(x, 0) = \varphi(x) \end{cases} \quad (3)$$

tenglamaning

$$U(x, t) = \frac{1}{2\pi} \int_R e^{ixz} \left[E_\alpha(-z^2 t^\alpha) + \int_0^t y^{\alpha-1} E_{\alpha,\alpha}(-z^2 y^\alpha) \hat{f}(z; t - y) dy \right] dz \quad (7)$$

yechimi klassik yechim bo'lishi kelib chiqadi. Demak, tenglamaning bu yechimidan boshqa shu tipdagi fizika-matematika tenglamalari yechishda foydalanish mumkin.

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