

Date: 3rd May-2025

FUNKSIYANING ANIQLANISH SOHASI, TOQLIGI VA JUFTLIGI, FUNKSIYANING CHEGARALANGANLIGI VA DAVRIYLIGI

Jumaboyeva Zevarjon Rustamboyevna

jumaboyevazevarjon9@gmail.com

Annotatsiya: Mazkur maqolada funksiyaning asosiy tushunchasi, uning matematik ifodalanishi, aniqlanish sohasi va asosiy xossalari tahlil qilinadi. Shuningdek, oddiy va murakkab funksiyalar, ularning grafiklari, limitlari, uzluksizligi hamda turlari — chiziqli, kvadratik, trigonometrik, eksponensial va logarifmik funksiyalar haqidagi nazariy ma'lumotlar yoritiladi. Mavzu matematik analizning tayanch bo'limi bo'lib, uni chuqur o'rganish matematik modellashtirish va amaliy hisoblashlarda muhim ahamiyat kasb etadi.
Kalit so'zlar: Funksiya, aniqlanish sohasi, qiymatlar sohasi, uzluksizlik, limit, chiziqli funksiya, kvadratik funksiya, trigonometrik funksiya, eksponensial funksiya, grafik, hosila, matematik analiz.

1-Ta'rif: Agar X to'plamdagi har bir x songa biror qoida yoki qonunga ko'ra Y to'plamdan bitta y son mos qo'yilsa, X to'plamda funksiya berilgan deyiladi va

$$f: x \rightarrow y \text{ yoki } y = f(x)$$

ko'rinishda yoziladi. Bunda X —funksiyaning aniqlanish sohasi, Y —funksiyaning o'zgarish sohasi deyiladi. Bu yerda x - erkli o'zgaruvchi (funksiya argumenti) y esa erksiz o'zgaruvchi (x o'zgaruvchining funksiyasi) deyiladi.

Har bir ratsional songa 1 ni, har bir irrotsionla songa 0 ni mos qo'yish natijasida funksiya hosil bo'ladi. Uni **Dirixle funksiyasi** deyiladi va $D(x)$ ko'rinishda belgilanadi va quyidagicha bo'adi:

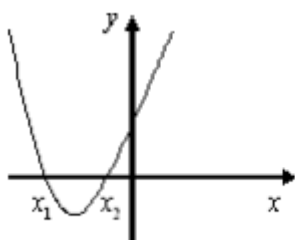
$$D(x) = \begin{cases} 1, & \text{agar } x \in \mathbb{R} \text{ ratsional son} \\ 0, & \text{agar } x \in \mathbb{R} \text{ irratsional son.} \end{cases}$$

Ta'rif: Agar $\forall x \in X$ uchun $f(-x) = f(x)$ bo'lsa $f(x)$ funksiya juft funksiya $f(-x) = -f(x)$ bo'lsa $f(x)$ funksiya toq funksiya deyiladi.

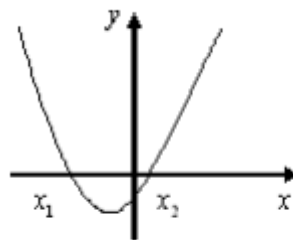
Asosiy elementar funksiyalar

1. **Parabola** $y = ax^2 + bx + c$ kvadrat tenglama, bu kvadrat tenglamaning grafigi a koeffitsient hamda diskriminant $D = b^2 - 4ac$ ning ishoralariga bog'liq va ularni quyidagicha keltirib o'tamiz:

$$1) \quad a > 0, D > 0, x_1 < 0, x_2 < 0 \quad 2) \quad a > 0, D > 0, x_1 < 0, x_2 > 0$$



Grafigi I,II,III choraklarda.

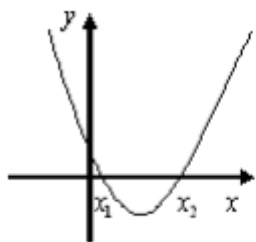


Grafigi I,II,III,IV choraklarda.

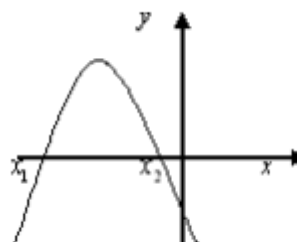
Date: 3rd May-2025

3) $a > 0, D > 0, x_1 > 0, x_2 > 0$

4) $a < 0, D > 0, x_1 < 0, x_2 < 0$



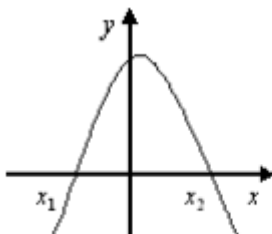
Grafi I, II, IV choraklarda.



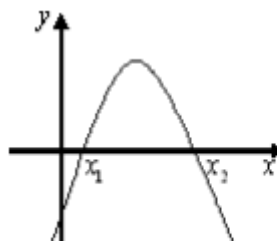
Grafi II, III, IV choraklarda.

5) $a < 0, D > 0, x_1 < 0, x_2 > 0$

6) $a < 0, D > 0, x_1 > 0, x_2 > 0$



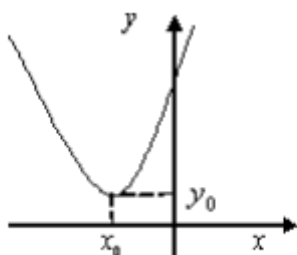
Grafi I, II, III, IV choraklarda.



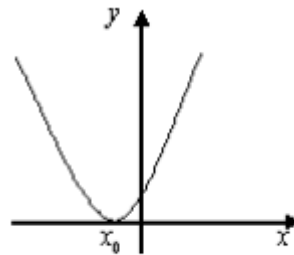
Grafi I, III, IV choraklarda.

7) $a > 0, D < 0, x_0 = -\frac{b}{2a}, y_0 = \frac{4ac-b^2}{4a} > 0$

8) $a > 0, D = 0, y = 0$



Grafi I, II choraklarda.



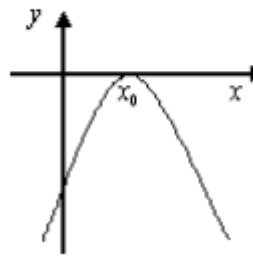
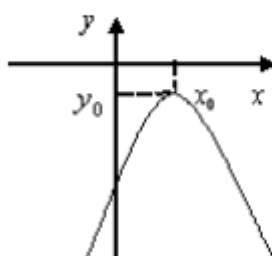
Grafi I, II choraklarda.

9) $a < 0, D < 0, x_0 = -\frac{b}{2a}, y_0 = \frac{4ac-b^2}{4a} < 0 > 0$

10) $a < 0, D = 0, y = 0$

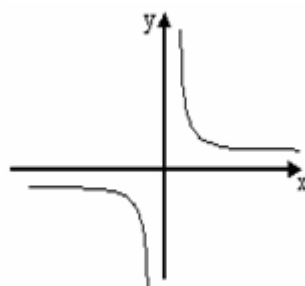
Grafi III, IV choraklarda.

Grafi III, IV choraklarda.



2. Giperbola $y = \frac{1}{x}$ funksiyaning grafi $x \neq 0$ qiymatlarida

3. Darajali

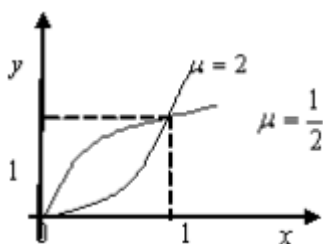


funksiya $y = x^n$

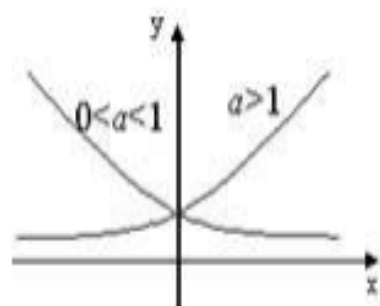


Date: 3rd May-2025

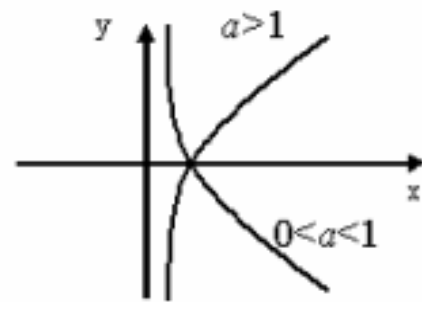
Darajali funksiyaning grafigi har doim tekislikning (0,0) hamda (1,1) nuqtasidan o'tadi.



4.Ko'rsatkichli funksiya $y = a^x$ ko'rinishdagi funksiya ko'rsatkichli funksiya deyiladi, bunda $a > 0$ va $a \neq 1$. Ko'rsatkichli funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plami, qiymatlar sohasi esa barcha musbat sonlar to'plami. Bu funksiyaning grafigi OX o'qidan o'tadi va $(0, 1)$ nuqtadan o'tadi.



5.Logarifmik funksiya $y = \log_a x$ ko'rinishidagi funksiya logarifmik funksiya deyiladi, $a > 0$ va $a \neq 1$. Bu funksiya $X = (0, +\infty)$ intervalda aniqlangan. Bu funksiyaning grafigi.



$y = f(x)$ funksiya X to'plamda berilgan bo'lsin.

2-Ta'rif: Agar shunday o'zgarmas M (o'zgarmas m) soni topilsaki, $\forall x \in X$ uchun

$$f(x) \leq M \quad (f(x) \geq m)$$

bo'lsa, $f(x)$ funksiya X to'plamda yuqoridan (quyidan) chegaralangan deb ataladi. Agar $f(x)$ funksiya ham yuqoridan ham quyidan chegaralangan bo'lsa, ya'ni shunday o'zgarmas M va m sonlar topilsaki, $\forall x \in X$ uchun

$$m \leq f(x) \leq M$$

bo'lsa, $f(x)$ funksiya X to'plamda chegaralangan deb ataladi.

3-Ta'rif: Agar $\forall x_1, x_2 \in X$ lar uchun $x_1 < x_2$ bo'lishidan $f(x_1) \leq f(x_2)$ ($f(x_1) < f(x_2)$) bo'lsa, $f(x)$ funksiya X to'plamda o'suvchi (qat'iy o'suvchi) deb ataladi.

Agar $\forall x_1, x_2 \in X$ lar uchun $x_1 < x_2$ bo'lishidan $f(x_1) \geq f(x_2)$ ($f(x_1) > f(x_2)$)

Date: 3rd May-2025

bo'lsa, $f(x)$ funksiya X to'plamda kamayuvchi (qat'iy kamayuvchi) deb ataladi. O'suvchi va kamayuvchi funksiya monoton funksiyalar deb ataladi.

4-Ta'rif: Agar shunday o'zgarmas T ($T \neq 0$) soni mavjud bo'lsaki, $\forall x \in X$ lar uchun

$$x + T \in X, \quad x - T \in X$$

$$f(x + T) = f(x)$$

bo'lsa, $f(x)$ funksiya davriy funksiya deyiladi va bu shartlarni qanoatlantiruvchi musbat T larning eng kichigi (agar u mavjud bo'lsa) funksiyaning davri deb ataladi.

5-Ta'rif: Agar $\forall x \in X$ va $-x \in X$ lar uchun $f(-x) = f(x)$ bo'lsa, $f(x)$ funksiya juft funksiya, $f(-x) = -f(x)$ bo'lsa, $f(x)$ funksiya toq funksiya deb ataladi.

Mavzu yuzasidan namunaviy misollar:

1-misol: Ushbu

$$y = \frac{1}{\sqrt{1-x^2}}$$

funksiyaning aniqlanish sohasini toping.

Yechish: $\sqrt{1-x^2}$ ifoda kasr maxrajida ekanligini hisobga olib $1-x^2 > 0$ munosabatga ega bo'lamiz. Bundan $|x| < 1$ demak, berilgan funksiyaning aniqlanish sohasi $(-1, 1)$ intervaldan iborat.

2-misol: Ushbu

$$f(x) = 2^{\cos^2 x} + 3 \sin 2x$$

funksiya chegaralanganligini ko'rsating.

Yechish: Bu funksiya $R = (-\infty, +\infty)$ da aniqlangan;

$$|2^{\cos^2 x} + 3 \sin 2x| \leq |2^{\cos^2 x}| + |3 \sin 2x| \leq 2 + 3 = 5$$

Demak, funksiya R chegaralangan.

3-misol: Ushbu

$$f(x) = x^3$$

funksiyaning o'suvchi ekanini isbotlang.

Yechish: Bunda $\forall x_1, x_2 \in R$ nuqtalarni olib $x_1 < x_2$ bo'lsin deb qaraylik. U holda

$$f(x_2) - f(x_1) = x_2^3 - x_1^3 = (x_2 - x_1)(x_2^2 + x_2 \cdot x_1 + x_1^2) =$$

$$(x_2 - x_1) \left[(x_2 + \frac{1}{2}x_1)^2 + \frac{3}{4}x_1^2 \right] > 0 \text{ bo'ladi.}$$

Demak, $\forall x_1, x_2 \in R, x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. Bu esa berilgan funksiyaning R da qat'iy o'suvchi ekanini bildiradi.

4-misol: Ushbu

$$f(x) = 3 \cdot \cos x + \cos 2x$$

funksiyaning davriy funksiya ekanini ko'rsating.

Yechish: Funksiyaning aniqlanish sohasi butun sonlar o'qidan iboratdir. Faraz qilaylik biror, $T > 0$ uchun

$$3 \cos(x + T) + \cos 2(x + T) = 3 \cos x + \cos 2x$$

munosabat o'rinli bo'lsin. $x = 0$ da

Date: 3rd May-2025

$$3\cos T + \cos 2T = 4$$

tenglamaga ega bo'lib, $\cos T \leq 1, \cos 2T \leq 1$ tengsizliklarni e'tiborga olsak,

$$3\cos T + \cos 2T \leq 4$$

bo'ladi.

Demak, T quyidagi tenglamalar sistemasini qanoatlantirdi:

$$\begin{cases} \cos T = 1 \\ \cos 2T = 1 \end{cases}$$

Bu tenglamalar sistemasining eng kichik musbat yechimi $T = 2\pi$ ekanini ravshandir.

Endi $T = 2\pi$ sonni berilgan funksiyaning davri ekanini tekshirish qiyin emas: $\forall x \in \mathbb{R}$ uchun

$$3\cos(x + 2\pi) + \cos 2(x + \pi) = 3\cos x + \cos 2x$$

tenglik o'rinlidir. Shunday qilib berilgan funksiya davriy bo'lib, ning davri 2π ga teng ekan.

5-misol: Ushbu

$$f(x) = \log_2(x + \sqrt{1 + x^2})$$

funksiyaning juft yoki toq funksiya ekanini aniqlang.

Yechish: $\forall x \in \mathbb{R}$ uchun $x + \sqrt{1 + x^2}$ bo'lgani uchun funksiyaning aniqlanish sohasi $\mathbb{R}$ dan iborat.

$\forall x \in \mathbb{R}$

uchun

$$\begin{aligned} f(-x) &= \log_2(-x + \sqrt{1 + x^2}) = \log_2 \frac{x + \sqrt{1 + x^2}}{x + \sqrt{1 + x^2}} \cdot (-x + \sqrt{1 + x^2}) = \\ &= \log_2 \frac{1}{x + \sqrt{1 + x^2}} = \log_2(x + \sqrt{1 + x^2})^{-1} = -\log_2(x + \sqrt{1 + x^2}) = -f(x) \end{aligned}$$

bo'ladi, bu esa qaralayotgan funksiyaning toq funksiya ekanini bildiradi.

6-Misol: Ushbu $y = f(x) = 2x + 1$ $x \in [0, 1]$ funksiyaning teskari funksiyasini toping.

Yechish: Bu funksiyaning qiymatlari to'plami $[1, 3]$ oraliqni tashkil etadi, bu oraliqda aniqlangan

FOYDALANILGAN ADABIYOTLAR:

1. Rasulov M.X. *Matematik analiz kursi*. – Toshkent: “Fan va texnologiya”, 2018.
2. Sultonov M.M., Tadjibayev R.R. *Matematik analiz*. – Toshkent: O'zbekiston milliy universiteti, 2016.
3. Kudarov M. *Matematik analizdan misollar va mashqlar*. – Samarqand: SamDU nashriyoti, 2020.
4. Apostol T.M. *Mathematical Analysis*. – Addison-Wesley, 1974.
5. Spivak M. *Calculus*. – Cambridge University Press, 2006.
6. Larson R., Edwards B.H. *Calculus of a Single Variable*. – Cengage Learning, 2013.
7. Stewart J. *Calculus: Early Transcendentals*. – Cengage Learning, 2015.
8. Zorich V.A. *Mathematical Analysis I & II*. – Springer, 2004.

